SPECTRA PRACTICE PAPER (2024-25)

SPECTRA CLASSES

Class – XII

Subject: Mathematics.

Time: 3hrs. M.M-80

General Instructions:

Read the following instructions very carefully and strictly follow them:

- This question paper contains **38** questions. **All** questions are **compulsory**.
- This question paper is divided into **five** Sections **A**, **B, C, D** and **E**.
- In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- In **Section D,** Questions no. **32** to **35** are long answer (LA) type questionscarrying **5** marks each.
- In **Section E,** Questions no. **36** to **38** are case study-based questions carrying **4** marks each.
- There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and2 questions in Section E.
- Use of calculators is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of **1 mark** each.

Q 1. If for a square matrix A, $A\,.\,adj\ A = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix}$, then the value of $|A| + |adj\ A|$ equal to:

a) 1 b) $2025 + 1$ c) $(2025)^2 + 45$ d) $(2025)^2 + 2025$

Q 2. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then find the value of $|3\ AB|$.

a) -80 b) 80 c) -81 d) 81

Q 3. Find the value of k, the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$?

a) $k = 2$ b) $k = -1$ c) $k = 1$ d) $k = -2$

Q 4. The value of $\int e^x(\log sinx + \cot x)dx$

a) $e^x \log \sin x + C$ b) $e^x \cot x + C$

c) $e^x \sin x + C$ d) $-e^x \sin x + C$

Q 5. If A and B are non-singular matrices of same order with $det(A) = 5$, then $\boldsymbol{det}(B^{-1}AB)^2$ is equal to

a) 5 b) 5^2 c) 5^4 d) 5^5

Q 6. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 + 7A$.

a) $15\ A$ b) $14\ A$ c) $12\ A$ d) $10\ A$

Q 7. $\int_0^{\frac{\pi}{2}} \frac{1}{2+\cos x}\, dx$

a) $\tan^{-1}\frac{1}{\sqrt{3}}$ b) $\frac{2}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}}$ c) $\sqrt{3}\tan^{-1}\sqrt{3}$ d) $2\sqrt{3}\tan^{-1}\sqrt{3}$

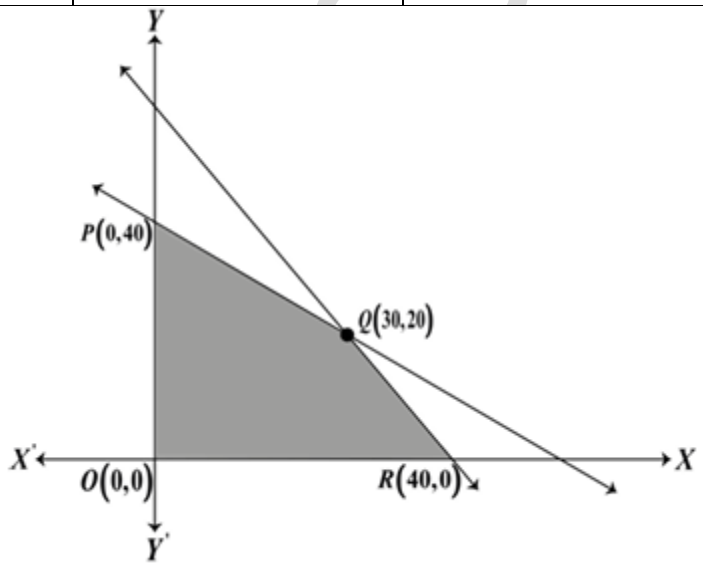
Q 8. If A and B are two events such that $P(A') = 0.6$, $P(B) = 0.8$ and $P(B \mid A) = 0.6$, then find $P(A \mid B)$

a) 0.4 b) 0.5 c) 0.6 d) 0.3

Q 9. If $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ is a skew-symmetric matrix then the value of $a + b + c =$

a) 1 b) 2 c) 3 d) 4

Q 10. For the linear programming problem (LPP), the objective function is $Z = 4x + 3y$ *and* the feasible region determined by a set of constraints is shown in the graph:



Which of the following statemnets is true?

a) Maximum value of Z is at $R(40,0)$.
b) Maximum value of Z is at $Q(30,20)$.
c) Value of Z at $R(40,0)$ is less tha the value at $P(0,40)$.
d) The value of Z at $Q(30,20)$ is less than the value at $R(40,0)$.

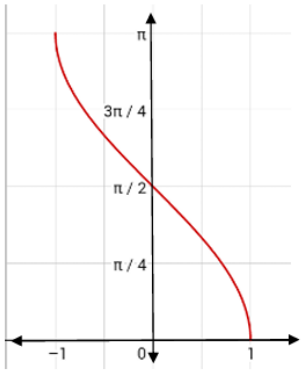
Q 11. $\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}}$ equlas

a) $-\frac{1}{2x^2}\sqrt{1+x^4}+C$ b) $\frac{1}{2x}\sqrt{1+x^4}+C$

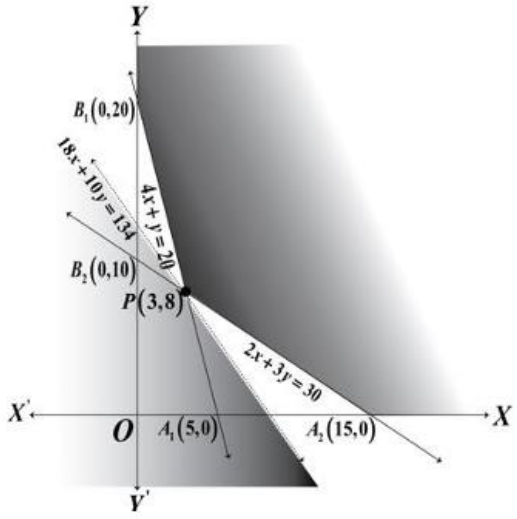
c) $-\frac{1}{4x}\sqrt{1+x^4}+C$ d) $-\frac{1}{4x^2}\sqrt{1+x^4}+C$

Q 12. The graph drawn below depicts

a) $y = \sin^{-1}x$
b) $y = \cos^{-1}x$
c) $y = cosec^{-1}x$
d) $y = \cot^{-1}x$



Q 13. A linear programming problem (LPP) along with the graph of its constraints is shown below. he corresponding objective function is: $Z = 18x + 10\ y$, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point (3, 8), The optimal solution of the above linear programming problem _____.



a) Does not exist as the feasible region is unbounded.

b) Does not exist as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

c) Exists as the inequality $18x + 10y > 134$ has infinitely many points in common with the feasible region.

d) Exists as the inequality $18x + 10y < 134$ does not have any point in common with feasible region.

Q 14. If $|\vec{a}| = 5, |\vec{b}| = 13, |\vec{a} \times \vec{b}| = 25$ *then* $\vec{a}.\vec{b}$ is equal to

a) 60	b) 25	c) 65	d) None

Q 15. For what value of K the matrix $\begin{bmatrix} 2K & -1 \\ -8 & 3 \end{bmatrix}$ is non invertible?

a) $\frac{2}{3}$	b) $\frac{1}{3}$	c) $\frac{4}{3}$	d) -1

Q 16.

i) $\frac{d}{dx}(\sin x)$ at $x = \frac{\pi}{2}$

ii) $\frac{d}{dx}(\tan^{-1} x)$ at $x = 1$

iii) $\frac{d}{dx}(e^x) at\ x = 0$

iv) $\frac{d}{dx}(x^x)$ at $x = e$

Arrangement of the above values in the increasing order of magnitude is

a) II, III, I, IV	b) IV, III, I, II	c) I, III, IV, II	d) I, II, III, IV

Q 17. If the order of the differential equation $\frac{d^3y}{dx^3} = 3 + \sqrt{\frac{dy}{dx}}$ is O and degree is D then find the value of $\frac{D}{O} + \frac{O}{D}$.

a) $\frac{10}{3}$	b) $\frac{13}{6}$	c) $\frac{13}{9}$	d) $\frac{10}{6}$

Q 18. Write the projection of the vector $\hat{\imath} - \hat{\jmath}$ on the vector $\hat{\imath} + \hat{\jmath}$

a) 0	b) 3	c) 2	d) 5

Questions number **19** and **20** are Assertion and Reason based questions carrying1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d)as given below.

a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).

b) Both Assertion(A) and Reason(R) are true and Reason(R) is not the correct explanation of the Assertion(A).

c) Assertion (A) is true, but Reason (R) is false.

d) Assertion (A) is false, but Reason (R) is true.

Q 19.	**Assertion (A):** consider the function defined as $f(x) = \|x\| + \|x-1\|, x \in R$. Then $f(x)$ is not differentiable at $x = 0\ and\ x = 1$. **Reason (R):** suppose f be defined and continuous on (a, b) and $c \in (a, b)$, then $f(x)$ is not differentiable at $x = c$ if $\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h} \neq \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$.
Q 20.	**Assertion (A):** the function $f: R - \left\{\frac{(2n+1)\pi}{2} : n \in Z\right\} \to (-\infty, -1] \cup [1, \infty)$ defined by $f(x) = \sec x$ is not one-one function in its domain. **Reason (R):** the line $y = 2$ meets the graph of the function at more than one point.

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks** each.

Q 21.	A function $f: A \to B$ defined as $f(x) = 2x$ is both one – one and onto if $A = \{1,2,3,4,\}$, then find the set B. OR Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1} 1$
Q 22.	Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.
Q 23.	Position vectors of the points A, B and C as shown in the figure below are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.  If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. **OR** Check whether the lines given by equations $x = 2t + 2, y = 7t + 1, z = -3t - 3$ and $x = -s - 2, y = 2s + 8, z = 4s + 5$ are perpendicular to each other or not.
Q 24.	If $y = \left(x + \sqrt{x^2 - 1}\right)^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.
Q 25.	Show that the function $f(x) = \frac{16\sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

SECTION C

This section comprises short answer (SA) type questions of **3 marks** each.

Q 26.	Evaluate: $\int_{-1}^{2} \|x^3 - x\|\ dx$

	OR Find: $\int \frac{\sqrt{x}dx}{\sqrt{a^3-x^3}}$.
Q 27.	Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$
Q 28.	Evaluate: $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$
Q 29.	Find the general solution of the differential equation: $(xy-x^2)dy = y^2dx$. **OR** Find the general solution of the differential equation: $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$.
Q 30.	Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also find the mean of the random variable. **OR** A and B throw a die alternately till one of them gets a $'6'$ and wins the game. Find their respective probabilities of winning, if A starts the game first.
Q 31.	Solve the following linear programming problem graphically: Minimize: $Z = 5x + 10y$ Subject to constraints: $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$.
	SECTION D This section comprises long answer type questions (LA) of **5 marks** each.
Q 32.	If $A = \begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}, B = \begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, then find AB and use it to solve the following system of equations: $x - 2y = 3$ $2x - y - z = 2$ $-2y + z = 3$ **OR** Find the inverse of the matrix $A = \begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations $x - y + 2z = 1;$ $2y - 3z = 1;$ $3x - 2y + 4z = 3.$
Q 33.	The area of the region included between the curve $4y = 3x^2$ and the line $2y = 3x + 12$
Q 34.	Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines. **OR**

	Find the equations of all the sides of the parallelogram ABCD whose vertices are $A(4,7,8), B(2,3,4), C(-1,-2,1)$ and $D(1,2,5)$. Also, find the coordinates of the foot of perpendicular from A to CD.
Q 35.	a) If $y = (\tan x)^x$, then find the $\frac{dy}{dx}$. b) If $y = (log\ x)^2$, prove that $x^2y'' + xy' = 2$.

SECTION E

This section comprises 3 case study-based questions of **4 marks** each.

Q 36. **Case Study – 1**

A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

a) Let $E_1\ and\ E_2$ respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1), P(E_2)$. 1

b) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$. 1

c) (i) Find the probability of customer paying second month's bill in time. 2

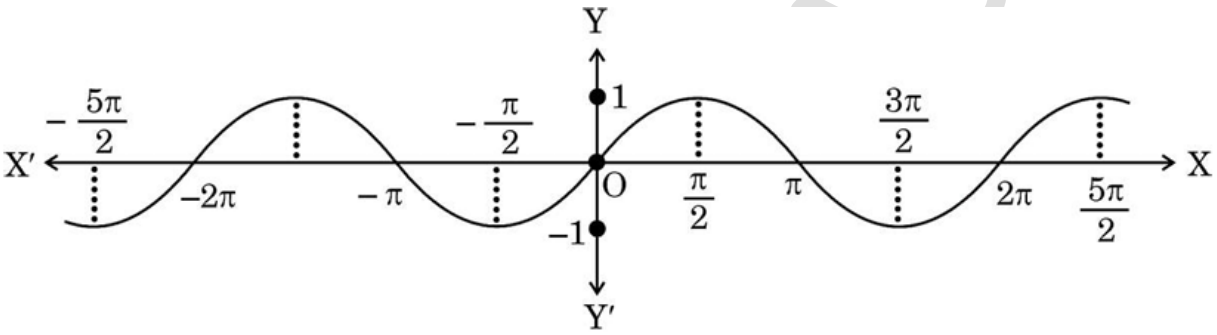
OR

(ii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time. 2

Q 37. **Case Study – 2**

Jatin, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension $25\ cm \times 40\ cm$ to make container packets without top. Let xcm be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.

Based on the above information answer the following questions.

	a) Express the volume (V) of each container as function of x only. 1 b) Find $\frac{dV}{dx}$. 1 c) For what value of x the volume of each container is maximum? 2 **OR** Check whether V has a point of inflection at $x = \frac{65}{6}$ or not? 2
Q 38.	**Case Study – 3** If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x), y \in Y$. Function g is called the inverse of function f. The domain of sine function is R and function $sine : R \to R$ is neither one-one nor onto. The following graph shows the sine function.  y = sin x Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A. On the basis of the above information, answer the following questions: a) If A is the interval other than principal value branch, give an example of one such interval. 1 b) If $\sin^{-1} x$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. 1 c) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. 2 **OR** Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. 2