SPECTRA PRACTICE PAPER (2025-26)
SPECTRA CLASSES
CLASS XIIth
SUBJECT: MATHEMATICS

Time: 3 hrs. **M. Marks :** 80

General Instructions

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion - Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) - type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) - type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) - type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1 Let R be any relation in the set A of human beings in a town at a particular time. If R ={ [x, y) : x is exactly 7 cm taller than y] , then R is **[1]**
a) an equivalence relation b) reflexive
c) not symmetric d) symmetric but not transitive

2 If $A = [cos\alpha\ sin\alpha\ \ -sin\alpha\ cos\alpha\ ]$, then **[1]**
a) A'A = I b) A'A = 2I c) A'A = 0 d) A'A = - I

3 Let A =$|0\ -1\ \ -1\ 0\ |$, then **[1]**
a) $A^2 = A$ b) $A^2 = 0$ c) $A^2 = I$ d) None fo these

4 If $X = [3\ \ -4\ 1\ \ -1\ ], B = [5\ 2\ \ -2\ 1\ ]\ \ A = [p\ q\ r\ s\ ]$ satisfy the equation AX = B, then the matrix A is equal to **[1]**
a) $[7\ 26\ 4\ 17\ ]$ b) $[-7\ \ -4\ 26\ 13\ ]$
c) $[-7\ 26\ \ -6\ 23\ ]$ d) $[-7\ 26\ 1\ \ -5\ ]$

5 If A, B are two n× n non - singular matrices, then what can you infer about AB? **[1]**
a) $(AB)^{-1}$ does not exist b) AB is non - singular
c) AB is singular d) $(AB)^{-1} = A^{-1} B^{-1}$

6 If y =$\tan^{-1} x + \cot^{-1} x + \sec^{-1} x + \text{cosec}^{-1} x$, then $\frac{dy}{dx}$ is equal to **[1]**
a) 1 b) $\frac{x^2-1}{x^2+1}$ c) 0 d) π

7 Let y be a function of x such that log (x + y) - 2xy = 0, then y'(0) is **[1]**

a) $\frac{1}{2}$ b) 1 c) $\frac{3}{2}$ d) 0

8 Evaluate: $\int \frac{(1-\cos x)\,dx}{\cos x(1+\cos x)}$ **[1]**

(a) $\log(\sec x + \tan x) - 2\tan(\frac{x}{2}) + C$

(b) $\log(\sec x - \tan x) - 2\tan(\frac{x}{2}) + C$

(c) $\log(\sec x + \tan x) + 2\tan(\frac{x}{2}) + C$

(d) None of these

9 The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[1]**

a) $\frac{5}{3}$ sq.units b) 1 sq. units c) $\frac{2}{3}$ $sq.units$ d) $\frac{1}{6}$ $sq.units$

10 The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to it at the point with the ordinate 3 and the x - axis is **[1]**

a) 6 b) None of these c) 7 d) 3

11 The number of solutions of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$, when y(1) = 2, is: **[1]**

a) infinite b) zero c) one d) two

12 The degree of the differential equation $(y'')^2 + (y')^3 = x\sin(y')$ is: **[1]**

a) 3 b) 1 c) 2 d) not defined

13 The magnitude of the scalar p for which the vector $p(-3\hat{\imath} - 2\hat{\jmath} + 13\hat{k})$ is of unit length is **[1]**

a) $\frac{1}{8}$ b) $\frac{1}{\sqrt{182}}$ c) $\sqrt{182}$ d) $\frac{1}{64}$

14 Find the direction cosines of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ **[1]**

a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ b) $-\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

c) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$ d) $\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

15 The projections of a line segment on X, Y and Z axes are 12, 4 and 3 respectively. The length and direction cosines of the line segment are **[1]**

a) $11; \frac{12}{11}, \frac{14}{11}, \frac{3}{11}$ b) $19; \frac{12}{19}, \frac{4}{19}, \frac{3}{19}$

c) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ d) $15; \frac{12}{15}, \frac{14}{15}, \frac{3}{15}$

16 The direction cosines of the line passing through the following points (- 2, 4, - 5),(1, 2, 3) is: **[1]**

a) $\frac{-3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ b) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

c) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$ d) $\frac{-3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$

17 A Linear Programming Problem is as follows: Minimize Z = 2x + y **[1]**
Subject to the constraints $x \geq 3$, $x \leq 9$, $y \geq 0$
$x - y \geq 0$, $x + y \leq 14$ The feasible region has
a) 5 corner points including (0, 0) and (9, 5)
b) 5 corner points including (3, 6) and (9, 5)
c) 5 corner points including (7, 7) and (3, 3)
d) 5 corner points including (14, 0) and (9, 0)

18 In an LPP, if the objective function z = ax + by has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is: **[1]**
a) 0 b) 2 c) infinite d) finite

19 Assertion (A): Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes. **[1]**
Reason (R): The acute angle between the lines x - 2 = 0 and $\sqrt{3x - y - 2}$ is 30^{o}.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20 Assertion (A): $\Delta = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$ where, A_{ij} is cofactor of a_{ij}. **[1]**
Reason (R): Δ = Sum of the products of elements of any row (or column) with their corresponding cofactors.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21 Examine the differentiability of f, where f is defined by $f(x) = \{1 + x,\ if\ x \leq 2\ \ 5 - x,\ if\ x > 2$ at x = 2. **[2]**

22 If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius. **[2]**

23 Evaluate: $\int \cos^4 x \sin^3 x\, dx$. **[2]**

24 Find the general solution of the differential equation: $(1 + x)\frac{dy}{dx} - y = e^{3x}(1 + x)^2$ **[2]**

25 Find the sum of the order and the degree of the differential equation: $\left(x + \frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$ **[2]**

26 If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then write the range of $|\lambda\hat{a}|$ **[2]**

Section C

27 Write down the magnitude of each of the following vectors: $\vec{c} = \left(\frac{1}{\sqrt{3}}\hat{\imath} - \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right)$. **[3]**

28 Let m be a given fixed positive integer. Let R = {(a, b): a, b∈ Z and (a - b) is divisible by m}.Show that R is an equivalence relation on Z. **[3]**

29 Find the values of a and b so that the function f(x) defined by $f(x) = \{x + a\sqrt{2}x sinx , if 0 \leq x < \pi/4 \; 2xcotx + b , if \pi/4 \leq x < \pi/2 \; acos2x - bsinx, if \pi/2 \leq x \leq \pi$ becomes continuous on **[3]**

30 Evaluate: $\frac{x^2}{\sqrt{1-x}} dx$ **[3]**

31 Using integration, find the area of the region: **[3]**
$\{(x, y) : 9x^2 + 4y^2 \leq 36, 3x + 2y \geq 6\}$

32 Solve the Linear Programming Problemgraphically: **[3]**
Maximize Z = 3x + 4y Subject to
$2x + 2y \leq 80$
$2x + 4y \leq 120$

33 Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black, if the first marble is not replaced before the second draw. **[3]**

Section D

34 **Read the following text carefully and answer the questions that follow:** **[4]**
Rajni is preparing for her board exams. So,she decided to prepare chart of formulas of maths and important facts related to each chapter, Forthe chapter Inverse Trigonometric Functions she has prepared the following table to

Inverse	Domain	Principal Value Branch
$\sin^{-1}$	[-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}$	[-1, 1]	$[0, \pi]$
$\operatorname{cosec}^{-1}$	R- (-1 , 1)	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1}$	R- (-1, 1)	$[0, \pi]$ - { $\frac{\pi}{2}$ \}
$\tan^{-1}$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}$	R	$(0, \pi)$

remember the principal branch values of inverse trigonometric functions.

1. Find the principal value of $\operatorname{cosec}^{-1}(-1)$. (1)
2. Find the principal value of $\sec^{-1}(-2)$. (1)
3. Solve the following equation: $tan^{-1}\left(\frac{x+1}{x-1}\right) + tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$. (2)

OR

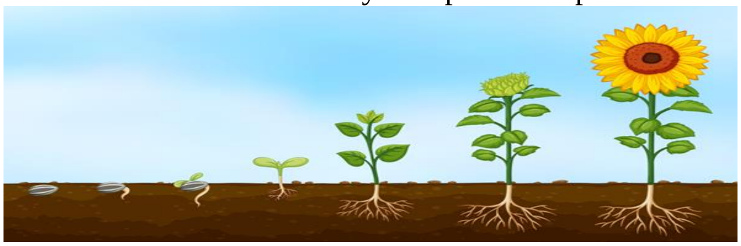
Show that $\sin^{-1} 2x\sqrt{1-x^2} = 2\sin^{-1} x$. (2)

35 In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield. **[4]**

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, 0 \le x \le 10$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions:

1. What are the critical points of the function f(x)? (2)
2. Using second derivative test, find the minimum value of the function. (2)

36 **Read the following text carefully and answer the questions that follow:** **[4]**

To hire a marketing manager, it’s important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

1. Find the probability that it is due to the appointment of Ajay (A). (1)
2. Find the probability that it is due to the appointment of Ramesh (B). (1)
3. Find the probability that it is due to the appointment of Ravi (C). (2)

OR

Find the probability that it is due to the appointment of Ramesh or Ravi. (2)

Section E

37 Using matrix method, solve the system of equations: **[5]**

$2x + y + z = 1, \quad x - 2y - z = \frac{3}{2}, \quad 3y - 5z = 9$

38 Evaluate $\int_0^{\frac{1}{2}} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (Hint: let x = sin θ) **[5]**

39 Find the shortest distance between the lines given by $\vec{r} = (8 + 3\lambda)\hat{\imath} - (9 + 16\lambda)\hat{\jmath} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$. **[5]**